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Diesel Engine Analytical Model

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Abstract— Modelling diesel engines is an effective tool to help in developing and assessing intelligent transportation systems and technologies as well as to help in predicting aggregate vehicle fuel consumption and emissions. Although vehicle analytical models are the vehicle modelling type that describes the physical phenomena associated with vehicle operation comprehensively based on the principles of physics with explainable mathematical trends, no analytical model has been developed as yet of diesel powertrains. This research paper presents an analytical model of a four-cylinder supercharged diesel engine as the heart of the diesel powertrain. This model serves to accurately analyze with explainable mathematical trends the performance of the supercharged diesel engine with respect to both of the transient response and steady state response.

Index Terms— Diesel Engine, Diesel Powertrain, Modelling, Intelligent Transportation Systems

1 INTRODUCTION

Diesel engines are among the largest contributors to air pollution. It has been reported that almost all populations living in developed countries are exposed frequently to diesel exhaust at some concentration and the potential for diesel exhaust to present a health hazard is proven [1]. Intelligent Transportation Systems (ITS) lie at the heart of the continual efforts of developing the diesel powertrains based on modeling in order to reduce the negative influence of diesel powertrains on the environment. Since the most contributing part of the diesel powertrain to diesel exhaust is the diesel engine [3], modelling diesel engines has received increasing attention. Among the seminal semi-analytical models of internal combustion engines is the Knock Integral Model (KIM) which was originally developed by Livengood and Wu, [4]. That model gives an implicit relation between the start of injection crankshaft angle, start of combustion crankshaft angle, and the physical in-cylinder parameters such as cylinder pressure, cylinder temperature, in-cylinder burned gas rate, and the fuel/air ratio. Metallidis and Natsiavas proposed another seminal semi-analytical model of the dynamics of single- and multi-cylinder reciprocating engines, which may involve torsional flexibility in the crankshaft [5]. They developed as well a linearized version of this model to acquire insight into some aspects of the system dynamics such as determining the steady-state response and investigating the effect of engine misfire. Ni, D., and Henclewood, D., presented and validated the Bernoulli model for vehicle infrastructure integration-enabled in-vehicle applications [6]. Although the model does not address in-cylinder gas flow dynamics, it presents an analytical relationship between air mass flow rate, engine power, and air state.

Other modeling techniques have been utilized as well in the modeling of internal combustion engines. Smits, [7] reviewed and compared the turbulence models for internal combustion engines and found that the k –ε turbulence empirical model is widely used because of its general applicability, robustness, and economy. That model consists of two empirical transport equations for the kinetic energy and dissipation rate. Divis, et al., [8] computationally modeled the diesel engine head using Finite Element modeling and thus described the thermal interaction and mechanical interaction between the several parts of the diesel engine head.

Although vehicle analytical models have the key advantage, over the other types of modelling described above, of describing the physical phenomena associated with vehicle operation comprehensively based on the principles of physics, with explainable mathematical trends, no comprehensive analytical model has been developed as yet of diesel powertrains. This research paper presents an analytical model of supercharged diesel engines, equipped with an electronic throttle control (ETC), as the heart of the diesel powertrain with the aim of helping in analyzing analytically the performance of the diesel engines.

2 DIESEL ENGINE ANALYTICAL MODEL

The main components of the diesel engine are timing belt, camshaft, cylinder head (valves), cylinder block, piston and connecting rod assembly, bearings and seals, and crankshaft. The timing belt links the crankshaft with the camshaft and makes the camshaft turns in the same direction of the crankshaft. The camshaft rotates at half the rotational speed of the crankshaft for four-cylinder diesel engines. This model addresses the dynamic interaction between the cylinder head (valves), cylinder block, piston and connecting rod assembly, and crankshaft. Figure 1, shows the schematic configuration of the diesel engine cylinder, piston, connecting rod, and crankshaft of supercharged diesel powertrain that is investigated in this research. In this section, the pressure ratios and temperature ratios are formulated analytically from the principles of physics for the compression stroke, combustion process, expansion stroke,

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and exhaust stroke of the diesel cycle that is shown on the p-v diagram on Fig. 2 and on the T-S diagram on Fig. 3. These formulations serve both chocked and non-chocked conditions as well as steady and transient conditions. In addition, the states throughout diesel cycle are defined analytically based on the principles of physics. Moreover, the in-cylinder gas speed dynamics are derived analytically from the principles of physics.

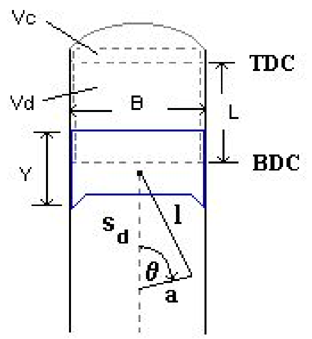

Fig. 1: Geometry of diesel engine cylinder, piston, connecting rod, and crankshaft [9]

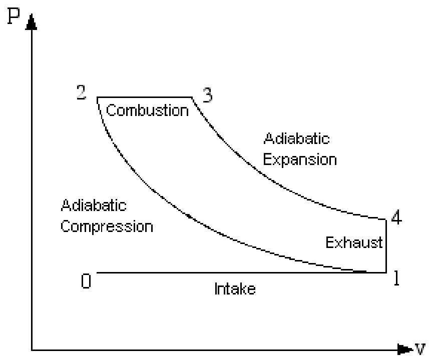

Fig. 2: Diesel cycle on P-V diagram [10]

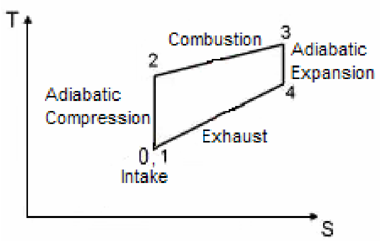

Fig. 3: Diesel cycle on T-S diagram [10]

2.1 Compression Stroke

In this section, the pressure ratios and temperature ratios for the strokes and processes of the diesel cycle are identified analytically from the principles of physics along with defining analytically from the principles of physics the states throughout the diesel cycle. The diesel cycle is shown on the p-v diagram on Fig. 2 and on the T-S diagram on Fig. 3. These formulations serve both chocked and non-chocked conditions as well as steady and transient conditions.

Since air flow is a gas flow, the following follows: 'Total energy equals the sum of the total energy associated with mass flow and work done.' Since, the total energy associated with mass flow comprises internal energy, kinetic energy, and potential energy, Therefore,

$$E_{Total} = \left(u + \frac{c^2}{2} + gz\right) + PV \tag{1}$$

Where:
E_{Total} is the total energy of the gas flow,
u is Internal energy,
c is the gas speed inside cylinder,
g is the gravitational acceleration,
z is the potential altitude.

Since by definition of the gas flow internal energy the following follows:

$$h = u + PV \tag{2}$$

Where:
h is the gas enthalpy.

Thus, combining Eq. (1) and (2) together leads to the following:

$$E_{Total} = h + \frac{c^2}{2} + gz \tag{3}$$

By applying the first law of thermodynamics on the conservation of energy to the in-cylinder compression process of the diesel cycle shown in Fig. 2 and Fig. 3, the following follows [11]:

$$h_1 + \frac{c_1^{\ 2}}{2} + gz_1 = h_2 + \frac{c_2^{\ 2}}{2} + gz_2 \tag{4}$$

Assuming the same altitude z, Eq. (4) can thus lead to the following:

$$h_1 + \frac{c_1^{\ 2}}{2} = h_2 + \frac{c_2^{\ 2}}{2} \tag{5}$$

Since at no phase change, at relatively high temperature, and/or at relatively low pressure air can be treated as an ideal gas and this is the case inside the cylinders of diesel engines, gas inside the cylinders of diesel engines can be treated as an ideal gas. Thus, it follows that:

$$h = C_P\ T \tag{6}$$

Where:
C_P is the gas specific heat at constant pressure, which is constant in the case of air is treated as an ideal gas.

Therefore, combining Eq. (5) and (6) leads to the following:

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$$C_P T_1 + \frac{c_1^{\;2}}{2} = C_P T_2 + \frac{c_2^{\;2}}{2} \tag{7}$$

Thus, Eq. (7) can be rewritten as:

$$T_1 = T_2 + \frac{\left(c_2^{\;2} - c_1^{\;2}\right)}{2\,C_P} \tag{8}$$

From the principles of the second and third laws of thermodynamics, it can be conceived that [11, 12]:

$$dS = \frac{\delta Q}{T} \tag{9}$$

Where:
S is the absolute entropy of the gas flow,
Q is heat flow.

For the compression stroke of the diesel cycle, the following follows from Eq. (9):

$$S_1 - S_2 = \int_2^1 \frac{\delta Q}{T} \tag{10}$$

Recalling the first law of thermodynamics, it follows that [11]:

$$\delta Q = dU + \delta W \tag{11}$$

By combining Eq. (10) and (11) together, the following follows:

$$S_1 - S_2 = \int_2^1 \frac{dU + dW}{T} \tag{12}$$

By definition of the work done, Eq. (12) can be rewritten as follows [11]:

$$S_1 - S_2 = \int_2^1 \frac{dU + (PdV + VdP)}{T} \tag{13}$$

By combining Eq. (2) and (13) together, the following follows:

$$S_1 - S_2 = \int_2^1 \frac{(dH - PdV) + (PdV + VdP)}{T} \tag{14}$$

Thus, simplifying Eq. (14) leads to the following:

$$S_1 - S_2 = \int_2^1 \frac{dH}{T} - \int_2^1 \frac{VdP}{T} \tag{15}$$

Combining the ideal gas law for air with the definition of density, it follows that [11]:

$$P = \rho R T \tag{16}$$

Where:
ρ is the density of air inside cylinder.

By combining Eq. (15) and (16) together along with the ideal gas law for air, the following follows [11]:

$$S_1 - S_2 = \int_2^1 \frac{dH}{T} - \int_2^1 \frac{mdP}{\rho T} \tag{17}$$

By combining Eq. (6) and (17) together along with the ideal gas law for air, the following follows [11]:

$$S_1 - S_2 = \int_2^1 C_P \frac{dT}{T} - \int_2^1 \frac{RdP}{P} \tag{18}$$

Hence, it follows mathematically from Eq. (18) that:

$$s_1 - s_2 = C_P \ln\left(\frac{T_1}{T_2}\right) - R \ln\left(\frac{P_1}{P_2}\right) \tag{19}$$

Where:
s is the specific entropy of the gas flow.

Therefore, rearranging Eq. (19) leads to the following:

$$\frac{s_1 - s_2}{R} = \frac{C_P}{R} \ln\left(\frac{T_1}{T_2}\right) - \ln\left(\frac{P_1}{P_2}\right) \tag{20}$$

From Eq. (2), it follows that:

$$dh = dU + d\,(PV) \tag{21}$$

By combining Eq. (21) with the ideal gas law for air, the following follows [11]:

$$dh = dU + RT \tag{22}$$

By dividing both sides of Eq. (22) by T and recalling Eq. (6) and recalling the similar equation to Eq. (6) for the specific heat at constant volume, it follows that [11]:

$$C_P = C_V + R \tag{23}$$

By the definition of the ratio of specific heat, k, Eq. (23) can be rewritten as [11]:

$$C_P = \frac{C_P}{k} + R \tag{24}$$

Thus, by rearranging Eq. (24) the following follows:

$$R = C_P\left(1 - \frac{1}{k}\right) \tag{25}$$

Hence, Eq. (25) can be rearranged as follows:

$$C_P = \frac{k\,R}{k-1} \tag{26}$$

By substituting Eq. (26) in Eq. (20):

$$\frac{s_1 - s_2}{R} = \frac{k}{k-1} \ln\left(\frac{T_1}{T_2}\right) - \ln\left(\frac{P_1}{P_2}\right) \tag{27}$$

Since the compression stroke is adiabatic by definition in the diesel cycle, the compression process can be assumed isentropic [13, 14]. Therefore, the following follows from Eq. (27):

$$\ln\left(\frac{T_1}{T_2}\right) = \frac{k-1}{k} \ln\left(\frac{P_1}{P_2}\right) \tag{28}$$

By establishing an inverse logarithmic operation on both sides of Eq. (28) the following mathematically follows:

$$\frac{T_1}{T_2} = \left(\frac{P_1}{P_2}\right)^{\frac{k-1}{k}} \tag{29}$$

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By substituting Eq. (26) in Eq. (8):

$$\frac{T_1}{T_2} = 1 + \frac{(k-1)\left(c_2^{\ 2} - c_1^{\ 2}\right)}{2\,k\,R\,T_2} \qquad (30)$$

Since Mach number, M, by definition is defined as:

$$M = \frac{c}{a} \qquad (31)$$

Where:
a is the speed of sound.

By definition, the speed of sound, a, is evaluated using the following formula [6, 12]:

$$a = \sqrt{k\,R\,T} \qquad (32)$$

Hence, by combining Eq. (30), (31), and (32) the following follows:

$$\frac{T_1}{T_2} = 1 + \frac{(k-1)}{2}\left(M^2 - \frac{c_1^{\ 2}}{k\,R\,T_2}\right) \qquad (33)$$

By rearranging Eq. (28), the following follows from Eq. (28):

$$\ln\left(\frac{P_1}{P_2}\right) = \frac{k}{k-1}\,\ln\left(\frac{T_1}{T_2}\right) \qquad (34)$$

By establishing an inverse logarithmic operation on both sides of Eq. (34), the following mathematically follows:

$$\frac{P_1}{P_2} = \left(\frac{T_1}{T_2}\right)^{\frac{k}{k-1}} \qquad (35)$$

Since the no significant heat transfer occurs in the cylinder during compression stroke, and the compression stroke is assumed to be isentropic as well, the following follows from combining Eq. (29) and (33):

$$\left(\frac{P_1}{P_2}\right)^{\frac{k-1}{k}} = 1 + \frac{(k-1)}{2}\left(M^2 - \frac{c_1^{\ 2}}{k\,R\,T_2}\right) \qquad (36)$$

By establishing a logarithmic operation on both sides of Eq. (36), the following mathematically follows:

$$\frac{k-1}{k}\ln\left(\frac{P_1}{P_2}\right) = \ln\left(1 + \frac{(k-1)}{2}\left(M^2 - \frac{c_1^{\ 2}}{k\,R\,T_2}\right)\right) \qquad (37)$$

By rearranging Eq. (37) and establishing an inverse logarithmic operation on both sides the following follows:

$$\frac{P_1}{P_2} = \left(1 + \frac{(k-1)}{2\,k\,R}\left(\frac{c_2^{\ 2}}{T_2} - \frac{c_1^{\ 2}}{T_2}\right)\right)^{\frac{k}{k-1}} \qquad (38)$$

By recalling the in-cylinder gas speed analytical formula from the research paper entitled "Diesel Powertrain Intake Manifold Analytical Model" [15] and combining this formula with Eq. (38), Thus:

$$\frac{P_1}{P_2} = \left(1 + \frac{(k-1)}{2\,k\,R\,T_2}\left(\left(\frac{\pi^4 B^4 r_C^{\ 2}\ N_m^{\ 2}\ a^2}{14400\ A_{0Mean}^{\ \ 2}}\,Sin^2\,\theta_2\right) - \left(\frac{\pi^4 B^4 r_C^{\ 2}\ N_m^{\ 2}\ a^2}{14400\ A_{0Mean}^{\ \ 2}}\,Sin^2\,\theta_1\right)\right)\right)^{\frac{k}{k-1}} \qquad (39)$$

Where:
B is cylinder bore diameter.
rc is the compression ratio in diesel engine.
N_m is crankshaft rotational speed (rev/min).
a is the crank length.
$A_{0\,Mean}$ is the mean cross sectional area of a frictionless throat at the entrance of the intake manifold.
θ_1 is the crankshaft angle of rotation at state 1 on diesel cycle.
θ_2 is the crankshaft angle of rotation at state 2 on diesel cycle.

The next process on diesel cycle is the combustion process. Hence, the temperature ratios in the combustion process of diesel cycle are derived analytically from the principles of physics in the following subsection.

2.2 Combustion Process

By applying the first law of thermodynamics on the conservation of energy to the in-cylinder combustion process of the diesel cycle shown in Fig. 2 and Fig. 3, the following follows [11]:

$$h_2 + \frac{c_2^{\ 2}}{2} + g z_2 = h_3 + \frac{c_3^{\ 2}}{2} + g z_3 \qquad (40)$$

Assuming the same altitude z, Eq. (40) can thus lead to the following:

$$h_2 + \frac{c_2^{\ 2}}{2} = h_3 + \frac{c_3^{\ 2}}{2} \qquad (41)$$

Therefore, combining Eq. (41) and (6) leads to the following:

$$C_P\,T_2 + \frac{c_2^{\ 2}}{2} = C_P\,T_3 + \frac{c_3^{\ 2}}{2} \qquad (42)$$

Thus, Eq. (42) can be rewritten as:

$$T_2 = T_3 + \frac{\left(c_3^{\ 2} - c_2^{\ 2}\right)}{2\,C_P} \qquad (43)$$

From the principles of the second and third laws of thermodynamics, it can be conceived for the combustion process in diesel cycle following from Eq. (9) that [11, 12]:

$$S_2 - S_3 = \int_3^2 \frac{\delta Q}{T} \qquad (44)$$

By combining Eq. (44) and (11) together, the following follows:

$$S_2 - S_3 = \int_3^2 \frac{dU + dW}{T} \qquad (45)$$

By definition of the work done, Eq. (45) can be rewritten as follows [11]:

$$S_2 - S_3 = \int_3^2 \frac{dU + (PdV + VdP)}{T} \qquad (46)$$

By combining Eq. (2) and (46) together, the following fol-

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lows:

$$S_2 - S_3 = \int_3^2 \frac{(dH - PdV) + (PdV + VdP)}{T} \tag{47}$$

Thus, simplifying Eq. (47) leads to the following:

$$S_2 - S_3 = \int_3^2 \frac{dH}{T} - \int_3^2 \frac{VdP}{T} \tag{48}$$

By combining Eq. (48) and (16) together along with the ideal gas law for air, the following follows [11]:

$$S_2 - S_3 = \int_3^2 \frac{dH}{T} - \int_3^2 \frac{mdP}{\rho T} \tag{49}$$

By combining Eq. (6) and (49) together along with the ideal gas law for air, the following follows [11]:

$$S_2 - S_3 = \int_3^2 C_P \frac{dT}{T} - \int_3^2 \frac{RdP}{P} \tag{50}$$

Hence, it follows mathematically from Eq. (50) that:

$$s_2 - s_3 = C_P \ln\left(\frac{T_2}{T_3}\right) - R \ln\left(\frac{P_2}{P_3}\right) \tag{51}$$

Therefore, rearranging Eq. (51) leads to the following:

$$\frac{s_2 - s_3}{R} = \frac{C_P}{R} \ln\left(\frac{T_2}{T_3}\right) - \ln\left(\frac{P_2}{P_3}\right) \tag{52}$$

By substituting Eq. (26) in Eq. (52):

$$\frac{s_2 - s_3}{R} = \frac{k}{k-1} \ln\left(\frac{T_2}{T_3}\right) - \ln\left(\frac{P_2}{P_3}\right) \tag{53}$$

Assuming frictionless combustion, Eq. (53) can thus be rewritten as:

$$\frac{\left(\frac{q_{i2} - q_{02}}{T_2}\right) - \left(\frac{q_{i3} - q_{03}}{T_3}\right)}{R} = \frac{k}{k-1} \ln\left(\frac{T_2}{T_3}\right) - \ln\left(\frac{P_2}{P_3}\right) \tag{54}$$

Where:

q_{i2} is the amount of heat exists in the cylinder just before reaching state 2 in diesel cycle.

q_{o2} is the amount of heat exists in the cylinder just after reaching state 2 in diesel cycle.

q_{i3} is the amount of heat exists in the cylinder just before reaching state 3 in diesel cycle.

q_{o3} is the amount of heat exists in the cylinder just after reaching state 3 in diesel cycle.

By rewriting Eq. (54) [12]:

$$\frac{\left(\frac{C_P (T_{i2} - T_2)}{T_2}\right) - \left(\frac{C_P (T_{i3} - T_3)}{T_3}\right)}{R} = \frac{k}{k-1} \ln\left(\frac{T_2}{T_3}\right) - \ln\left(\frac{P_2}{P_3}\right) \tag{55}$$

Where:

T_{i2} is the gas temperature just before the beginning of the combustion process in diesel cycle.

T_{i3} is the gas temperature just after the end of the combustion process in diesel cycle.

Therefore, the following follows from Eq. (55):

$$\frac{C_P}{R}\left(\frac{T_{i2}}{T_2} - \frac{T_{i3}}{T_3}\right) = \frac{k}{k-1} \ln\left(\frac{T_2}{T_3}\right) - \ln\left(\frac{P_2}{P_3}\right) \tag{56}$$

Since the combustion process in diesel cycle is isobaric, the following follows by establishing an inverse logarithmic operation on Eq. (56):

$$\frac{T_2}{T_3} = e^{\frac{C_P (k-1)}{Rk}\left(\frac{T_{i2}}{T_2} - \frac{T_{i3}}{T_3}\right)} \tag{57}$$

By combining Eq. (26) and (43):

$$\frac{T_2}{T_3} = 1 + \frac{k-1}{2} \frac{\left(c_3^{\ 2} - c_2^{\ 2}\right)}{k R T_3} \tag{58}$$

By combining Eq. (31), (32), and (58):

$$\frac{T_2}{T_3} = 1 + \frac{k-1}{2}\left(M^2 - \frac{c_2^{\ 2}}{k R T_3}\right) \tag{59}$$

By combining Eq. (57) and (59):

$$\ln\left(1 + \frac{k-1}{2}\left(M^2 - \frac{c_2^{\ 2}}{k R T_3}\right)\right) = \frac{C_P (k-1)}{T_2 R k}\left(T_{i2} - T_{i3}\left(\frac{T_2}{T_3}\right)\right) \tag{60}$$

Equation (60) can be rewritten as follows:

$$\left(\frac{T_2}{T_3}\right) = \frac{T_{i2}}{T_{i3}} - \frac{T_2 R k}{T_{i3} C_P (k-1)} \ln\left(1 + \frac{k-1}{2}\left(M^2 - \frac{c_2^{\ 2}}{k R T_3}\right)\right) \tag{61}$$

By recalling the in-cylinder gas speed analytical formula from the research paper entitled "Diesel Powertrain Intake Manifold Analytical Model" [15] and combining this formula with Eq. (61), Thus:

$$\left(\frac{T_2}{T_3}\right) = \frac{T_{i2}}{T_{i3}} - \frac{T_2 R k}{T_{i3} C_P (k-1)} \ln\left(1 + \frac{k-1}{2}\left(M^2 - \frac{\pi^4 B^4 r_C^{\ 2} N_m^{\ 2} a^2}{14400\, k R T_3 A_{0Mean}^{\ 2}} Sin^2 \theta_2\right)\right) \tag{62}$$

The expansion stroke happens next to the combustion process in diesel cycle. Thus, the pressure ratios in the expansion stroke of diesel cycle are derived analytically from the principles of physics in the following subsection.

2.3 Expansion Stroke

By applying the first law of thermodynamics on the conservation of energy to the in-cylinder expansion process of the diesel cycle shown in Fig. 2 and Fig. 3, the following follows [11]:

$$h_3 + \frac{c_3^{\ 2}}{2} + g z_3 = h_4 + \frac{c_4^{\ 2}}{2} + g z_4 \tag{63}$$

Assuming the same altitude z, Eq. (63) can thus lead to the following:

$$h_3 + \frac{c_3^{\ 2}}{2} = h_4 + \frac{c_4^{\ 2}}{2} \tag{64}$$

Therefore, combining Eq. (6) and (64) leads to the following:

$$C_P T_3 + \frac{c_3^{\ 2}}{2} = C_P T_4 + \frac{c_4^{\ 2}}{2} \tag{65}$$

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Thus, Eq. (65) can be rewritten as:

$$T_3 = T_4 + \frac{\left(c_4^{\ 2} - c_3^{\ 2}\right)}{2\,C_P} \tag{66}$$

For the expansion stroke of the diesel cycle, the following follows from Eq. (9):

$$S_3 - S_4 = \int_4^3 \frac{\delta Q}{T} \tag{67}$$

By combining Eq. (67) and (11) together, the following follows:

$$S_3 - S_4 = \int_4^3 \frac{dU + dW}{T} \tag{68}$$

By definition of the work done, Eq. (68) can be rewritten as follows [11]:

$$S_3 - S_4 = \int_4^3 \frac{dU + (PdV + VdP)}{T} \tag{69}$$

By combining Eq. (2) and (69) together, the following follows:

$$S_3 - S_4 = \int_4^3 \frac{(dH - PdV) + (PdV + VdP)}{T} \tag{70}$$

Thus, simplifying Eq. (70) leads to the following:

$$S_3 - S_4 = \int_4^3 \frac{dH}{T} - \int_4^3 \frac{VdP}{T} \tag{71}$$

By combining Eq. (71) and (16) together along with the ideal gas law for air, the following follows [11]:

$$S_3 - S_4 = \int_4^3 \frac{dH}{T} - \int_4^3 \frac{mdP}{\rho T} \tag{72}$$

By combining Eq. (6) and (72) together along with the ideal gas law for air, the following follows [11]:

$$S_3 - S_4 = \int_4^3 C_P \frac{dT}{T} - \int_4^3 \frac{RdP}{P} \tag{73}$$

Hence, it follows mathematically from Eq. (73) that:

$$s_3 - s_4 = C_P \ln\left(\frac{T_3}{T_4}\right) - R \ln\left(\frac{P_3}{P_4}\right) \tag{74}$$

Therefore, rearranging Eq. (74) leads to the following:

$$\frac{s_3 - s_4}{R} = \frac{C_P}{R} \ln\left(\frac{T_3}{T_4}\right) - \ln\left(\frac{P_3}{P_4}\right) \tag{75}$$

By substituting Eq. (26) in Eq. (75):

$$\frac{s_3 - s_4}{R} = \frac{k}{k-1} \ln\left(\frac{T_3}{T_4}\right) - \ln\left(\frac{P_3}{P_4}\right) \tag{76}$$

Since the expansion stroke is adiabatic by definition in the diesel cycle, the expansion process can be assumed isentropic [13, 14]. Therefore, the following follows from Eq. (76):

$$\ln\left(\frac{T_3}{T_4}\right) = \frac{k-1}{k} \ln\left(\frac{P_3}{P_4}\right) \tag{77}$$

By establishing an inverse logarithmic operation on both sides of Eq. (28) the following mathematically follows:

$$\frac{T_3}{T_4} = \left(\frac{P_3}{P_4}\right)^{\frac{k-1}{k}} \tag{78}$$

By substituting Eq. (26) in Eq. (66):

$$\frac{T_3}{T_4} = 1 + \frac{(k-1)\left(c_4^{\ 2} - c_3^{\ 2}\right)}{2\,k\,R\,T_4} \tag{79}$$

Hence, by combining Eq. (79), (31), and (32) the following follows:

$$\frac{T_3}{T_4} = 1 + \frac{(k-1)}{2}\left(M^2 - \frac{c_3^{\ 2}}{k\,R\,T_4}\right) \tag{80}$$

By rearranging Eq. (77), the following follows:

$$\ln\left(\frac{P_3}{P_4}\right) = \frac{k}{k-1} \ln\left(\frac{T_3}{T_4}\right) \tag{81}$$

By establishing an inverse logarithmic operation on both sides of Eq. (81), the following mathematically follows:

$$\frac{P_3}{P_4} = \left(\frac{T_3}{T_4}\right)^{\frac{k}{k-1}} \tag{82}$$

Since the no significant heat transfer occurs in the cylinder during compression stroke, and the compression stroke is assumed to be isentropic as well, the following follows from combining Eq. (80) and (82):

$$\left(\frac{P_3}{P_4}\right)^{\frac{k-1}{k}} = 1 + \frac{(k-1)}{2}\left(M^2 - \frac{c_3^{\ 2}}{k\,R\,T_4}\right) \tag{83}$$

By establishing a logarithmic operation on both sides of Eq. (83), the following mathematically follows:

$$\frac{k-1}{k} \ln\left(\frac{P_3}{P_4}\right) = \ln\left(1 + \frac{(k-1)}{2}\left(M^2 - \frac{c_3^{\ 2}}{k\,R\,T_4}\right)\right) \tag{84}$$

By rearranging Eq. (84) and establishing an inverse logarithmic operation on both sides the following follows:

$$\frac{P_3}{P_4} = \left(1 + \frac{(k-1)}{2\,k\,R}\left(\frac{c_4^{\ 2}}{T_4} - \frac{c_3^{\ 2}}{T_4}\right)\right)^{\frac{k}{k-1}} \tag{85}$$

By recalling the in-cylinder gas speed analytical formula from the research paper entitled “Diesel Powertrain Intake Manifold Analytical Model” [15] and combining this formula with Eq. (85), Thus:

$$\frac{P_3}{P_4} = \left(1 + \frac{(k-1)}{2\,k\,R\,T_4}\left(\left(\frac{\pi^4\,B^4\,r_C^{\ 2}\ N_m^{\ 2}\ a^2}{14400\ A_{0Mean}^{\ \ 2}}\,Sin^2\,\theta_4\right) - \left(\frac{\pi^4\,B^4\,r_C^{\ 2}\ N_m^{\ 2}\ a^2}{14400\ A_{0Mean}^{\ \ 2}}\,Sin^2\,\theta_3\right)\right)\right)^{\frac{k}{k-1}} \tag{86}$$

Where:

θ_3 is the crankshaft angle of rotation at state 3 on diesel cycle.
θ_4 is the crankshaft angle of rotation at state 4 on diesel cycle.

The last stroke on diesel cycle is the exhaust stroke. The temperature ratios in the exhaust stroke of diesel cycle are derived analytically from the principles of physics in the following subsection.

2.4 Exhaust Stroke

By applying the first law of thermodynamics on the conservation of energy to the in-cylinder exhaust process of the diesel cycle shown in Fig. 2 and Fig. 3, the following follows [11]:

$$h_4 + \frac{c_4^{\ 2}}{2} + gz_4 = h_1 + \frac{c_1^{\ 2}}{2} + gz_1 \tag{87}$$

Assuming the same altitude z, Eq. (87) can thus lead to the following:

$$h_4 + \frac{c_4^{\ 2}}{2} = h_1 + \frac{c_1^{\ 2}}{2} \tag{88}$$

Therefore, combining Eq. (88) and (6) leads to the following:

$$C_P\, T_4 + \frac{c_4^{\ 2}}{2} = C_P\, T_1 + \frac{c_1^{\ 2}}{2} \tag{89}$$

Thus, Eq. (89) can be rewritten as:

$$T_4 = T_1 + \frac{\left(c_1^{\ 2} - c_4^{\ 2}\right)}{2\,C_P} \tag{90}$$

From the principles of the second and third laws of thermodynamics, it can be conceived for the combustion process in diesel cycle following from Eq. (9) that [11, 12]:

$$S_4 - S_1 = \int_1^4 \frac{\delta Q}{T} \tag{91}$$

By combining Eq. (91) and (11) together, the following follows:

$$S_4 - S_1 = \int_1^4 \frac{dU + dW}{T} \tag{92}$$

By definition of the work done, Eq. (92) can be rewritten as follows [11]:

$$S_4 - S_1 = \int_1^4 \frac{dU + (PdV + VdP)}{T} \tag{93}$$

By combining Eq. (2) and (93) together, the following follows:

$$S_4 - S_1 = \int_1^4 \frac{(dH - PdV) + (PdV + VdP)}{T} \tag{94}$$

Thus, simplifying Eq. (94) leads to the following:

$$S_4 - S_1 = \int_1^4 \frac{dH}{T} - \int_1^4 \frac{VdP}{T} \tag{95}$$

By combining Eq. (95) and (16) together along with the ideal gas law for air, the following follows [11]:

$$S_4 - S_1 = \int_1^4 \frac{dH}{T} - \int_1^4 \frac{mdP}{\rho T} \tag{96}$$

By combining Eq. (6) and (96) together along with the ideal gas law for air, the following follows [11]:

$$S_4 - S_1 = \int_1^4 C_P \frac{dT}{T} - \int_1^4 \frac{RdP}{P} \tag{97}$$

Hence, it follows mathematically from Eq. (97) that:

$$s_4 - s_1 = C_P \ln\left(\frac{T_4}{T_1}\right) - R \ln\left(\frac{P_4}{P_1}\right) \tag{98}$$

Therefore, rearranging Eq. (98) leads to the following:

$$\frac{s_4 - s_1}{R} = \frac{C_P}{R} \ln\left(\frac{T_4}{T_1}\right) - \ln\left(\frac{P_4}{P_1}\right) \tag{99}$$

By substituting Eq. (26) in Eq. (99):

$$\frac{s_4 - s_1}{R} = \frac{k}{k-1} \ln\left(\frac{T_4}{T_1}\right) - \ln\left(\frac{P_4}{P_1}\right) \tag{100}$$

Assuming frictionless combustion, Eq. (100) can thus be rewritten as:

$$\frac{\left(\frac{q_{i4} - q_{04}}{T_4}\right) - \left(\frac{q_{i1} - q_{01}}{T_1}\right)}{R} = \frac{k}{k-1} \ln\left(\frac{T_4}{T_1}\right) - \ln\left(\frac{P_4}{P_1}\right) \tag{101}$$

Where:
q_{i4} is the amount of heat exists in the cylinder just before reaching state 4 in diesel cycle.
q_{o4} is the amount of heat exists in the cylinder just after reaching state 4 in diesel cycle.
q_{i1} is the amount of heat exists in the cylinder just before reaching state 1 in diesel cycle.
q_{o1} is the amount of heat exists in the cylinder just after reaching state 1 in diesel cycle.

By rewriting Eq. (101) [12]:

$$\frac{\left(\frac{C_P\,(T_{i4} - T_4)}{T_4}\right) - \left(\frac{C_P\,(T_{i1} - T_1)}{T_1}\right)}{R} = \frac{k}{k-1} \ln\left(\frac{T_4}{T_1}\right) - \ln\left(\frac{P_4}{P_1}\right) \tag{102}$$

Where:
T_{i4} is the gas temperature just before the beginning of the exhaust process in diesel cycle.
T_{i1} is the gas temperature just after the end of the exhaust process in diesel cycle.

Therefore, the following follows from Eq. (102):

$$\frac{C_P}{R}\left(\frac{T_{i4}}{T_4} - \frac{T_{i1}}{T_1}\right) = \frac{k}{k-1} \ln\left(\frac{T_4}{T_1}\right) - \ln\left(\frac{P_4}{P_1}\right) \tag{103}$$

Since the exhaust process in diesel cycle is isochoric, the following follows by applying the ideal gas law to Eq.

(103):

$$\frac{C_P}{R}\left(\frac{T_{i4}}{T_4}-\frac{T_{i1}}{T_1}\right)=\frac{k}{k-1}\ln\left(\frac{T_4}{T_1}\right)-\ln\left(\frac{T_4}{T_1}\right) \tag{104}$$

Thus, the following follows by establishing an inverse logarithmic operation on Eq. (104):

$$\frac{T_4}{T_1}=e^{\frac{C_P(k-1)}{R}\left(\frac{T_{i4}}{T_4}-\frac{T_{i1}}{T_1}\right)} \tag{105}$$

By combining Eq. (26) and (90):

$$\frac{T_4}{T_1}=1+\frac{k-1}{2}\frac{\left(c_1^{\,2}-c_4^{\,2}\right)}{k\,R\,T_1} \tag{106}$$

By combining Eq. (31), (32), and (106):

$$\frac{T_4}{T_1}=1+\frac{k-1}{2}\left(M^2-\frac{c_4^{\,2}}{k\,R\,T_1}\right) \tag{107}$$

By combining Eq. (105) and (107):

$$\ln\left(1+\frac{k-1}{2}\left(M^2-\frac{c_4^{\,2}}{k\,R\,T_1}\right)\right)=\frac{C_P(k-1)}{T_4\,R}\left(T_{i4}-T_{i1}\left(\frac{T_4}{T_1}\right)\right) \tag{108}$$

Equation (108) can be rewritten as follows:

$$\left(\frac{T_4}{T_1}\right)=\frac{T_{i4}}{T_{i1}}-\frac{T_4\,R}{T_{i1}\,C_P\,(k-1)}\ln\left(1+\frac{k-1}{2}\left(M^2-\frac{c_4^{\,2}}{k\,R\,T_1}\right)\right) \tag{109}$$

By recalling the in-cylinder gas speed analytical formula from the research paper entitled “Diesel Powertrain Intake Manifold Analytical Model” [15] and combining this formula with Eq. (109), Thus:

$$\left(\frac{T_4}{T_1}\right)=\frac{T_{i4}}{T_{i1}}-\frac{T_4\,R}{T_{i1}\,C_P\,(k-1)}\ln\left(1+\frac{k-1}{2}\left(M^2-\frac{\pi^4\,B^4\,r_C^{\,2}\,N_m^{\,2}\,a^2}{14400\,k\,R\,T_1\,A_{0Mean}^{\,2}}\,Sin^2\,\theta_4\right)\right) \tag{110}$$

Where:
θ_4 is the crankshaft angle of rotation at state 4 on diesel cycle.

Therefore, the states throughout diesel cycle can be determined analytically based on the principles of physics. The following subsection presents this.

2.5 Determining the States Analytically Throughout Diesel Cycle

The four states in the diesel cycle can be determined analytically using the principles of physics. The first state among them is state 1, which is determined analytically as follows:

$$P_1=P_{Compressor} \tag{111}$$

Where:
$P_{Compressor}$ is the pressure at the outlet of the supercharging compressor as stated in the compressor map corresponding to the maximum efficiency of the compressor and the required air flow rate.

$$T_1=T_i=\frac{T_{Intercooler}}{\eta_{Intercooler}} \tag{112}$$

Where:
T_i is the intake manifold temperature.
$T_{Intercooler}$ is the temperature at the outlet of the intercooler as stated in the intercooler catalogue.
$\eta_{Intercooler}$ is the intercooler efficiency which depends on the range of temperature of inlet air flow and is stated in the intercooler catalogue.

The intercooler efficiency is analytically formulated as follows:

$$\eta_{Intercooler}=\frac{T_{Inlet-Intercooler}-T_{Intercooler}}{T_{Inlet-Intercooler}-T_{Ref-Amb}} \tag{113}$$

Where:
$T_{Inlet-Intercooler}$ is the intercooler inlet temperature (or post compressor temperature).
$T_{Ref-Amb}$ is the ambient reference temperature.

By combining the equation of state for ideal gases, i.e. Eq. (16), along with Eq. (111) and (112), the volume of the gas can be determined as well analytically:

$$v_1=\frac{R\,T_{Intercooler}}{\eta_{Intercooler}\,P_{Compressor}} \tag{114}$$

Where:
v_1 is the specific volume of gas at state 1.

The second state in the diesel cycle can be determined analytically as well following from the principles of physics. State 2 on diesel cycle can be analytically formulated as follows:

$$P_2=P_{Peak} \tag{115}$$

Where:
P_{Peak} is the peak pressure inside cylinders which is commercially set in the diesel engine catalogue for each category of diesel engines.

In order to determine T_2, let us recall Eq. (29) for the compression stroke:

$$\frac{P_{Compressor}}{P_{Peak}}=\left(\frac{T_1}{T_2}\right)^{\frac{k}{k-1}} \tag{116}$$

By combining Eq. (116) and (112):

$$T_2=\frac{k\;T_{Intercooler}}{\eta_{Intercooler}\,(k-1)\,\ln\left(\frac{P_{Compressor}}{P_{Peak}}\right)} \tag{117}$$

By combining Eq. (16), (115), and (117):

$$v_2=\frac{R\;k\;T_{Intercooler}}{\eta_{Intercooler}\,P_{Peak}\,(k-1)\,\ln\left(\frac{P_{Compressor}}{P_{Peak}}\right)} \tag{118}$$

Where:
v_2 is the specific volume of gas at state 2.

By applying Eq. (16) to each of states 1 and 4 of the constant volume exhaust stroke in diesel cycle and dividing the results, the following follows:

$$\frac{P_{Compressor}}{P_4}=\frac{T_1}{T_4} \quad (119)$$

By combining Eq. (119), Eq. (112), and Eq. (82) for isentropic expansion stroke, the following follows:

$$\frac{P_{Compressor}}{\left(\frac{P_3}{\left(\frac{T_3}{T_4}\right)^{\frac{k}{k-1}}}\right)}=\frac{T_{Intercooler}}{\eta_{Intercooler}\, T_4} \quad (120)$$

Since the combustion process in diesel cycle is isobaric, the following thus follows from the p-v diagram of diesel cycle on Fig. 2:

$$P_3 = P_{Peak} \quad (121)$$

The following follows as well from the T-S diagram of diesel cycle on Fig. 3:

$$T_3 = T_{Peak} \quad (122)$$

Where:

T_{Peak} is the peak temperature inside cylinders which is commercially set in the diesel engine catalogue for each category of diesel engines.

Hence, by combining Eq. (120), (121), and (122), the following follows mathematically from Eq. (120) that:

$$T_4=\frac{\eta_{Intercooler}\, P_{Compressor}\, \left(T_{Peak}\right)^{\frac{k}{k-1}}}{T_{Intercooler}\, P_{Peak}} \quad (123)$$

Therefore, by substituting Eq. (123) in Eq. (82) for isentropic expansion stroke:

$$P_4=\left(\frac{T_{Peak}}{P_{Peak}}\right)^{\frac{1}{k-1}}\left(\frac{T_{Intercooler}}{\eta_{Intercooler}\, P_{Compressor}}\right)^{\frac{k}{1-k}} \quad (124)$$

In order to determine v_3 , combining Eq. (16) and Eq. (82) for isentropic expansion stroke leads to the following:

$$\frac{P_{Peak}}{P_4}=\left(\frac{P_{Peak}\, v_3}{P_4\, v_4}\right)^{\frac{k}{k-1}} \quad (125)$$

Where:

v_3 is the specific volume of gas at state 3.

v_4 is the specific volume of gas at state 4.

Since the exhaust stroke in diesel cycle is isochoric, the following thus follows from Eq. (114):

$$v_4 = v_1 = \frac{R\, T_{Intercooler}}{\eta_{Intercooler}\, P_{Compressor}} \quad (126)$$

Therefore, the following mathematically follows from Eq. (125):

$$\left(\frac{P_{Peak}}{P_4}\right)^{\frac{-1}{k-1}}=\left(\frac{v_3}{v_1}\right)^{\frac{k}{k-1}} \quad (127)$$

By combining Eq. (127) and (16):

$$\frac{P_{Peak}}{P_4}=\left(\frac{P_{Peak}\, v_3}{P_4\, v_4}\right)^{\frac{k}{k-1}} \quad (128)$$

Hence, the following mathematically follows from Eq. (128):

$$v_3 = v_1\left(\frac{P_{Peak}}{P_4}\right)^{\frac{-1}{k}} \quad (129)$$

Thus, combining Eq. (129), (114), and (124) leads to the following:

$$v_3=\left(\frac{R\, T_{Intercooler}}{\eta_{Intercooler}\, P_{Compressor}}\right)\left(\frac{P_{Peak}}{\left(\frac{T_{Peak}}{P_{Peak}}\right)^{\frac{1}{k-1}}\left(\frac{T_{Intercooler}}{\eta_{Intercooler}\, P_{Compressor}}\right)^{\frac{k}{1-k}}}\right)^{\frac{-1}{k}} \quad (130)$$

Hence, the following mathematically follows from Eq. (130):

$$v_3=\frac{R\left(\frac{\eta_{Intercooler}\, P_{Compressor}}{T_{Intercooler}}\right)^{\frac{k-2}{1-k}}}{\left(P_{Peak}\left(T_{Peak}\right)^{\frac{1}{k}}\right)^{\frac{1}{k-1}}} \quad (131)$$

In an endeavour to cover the key parameters in diesel powertrains in this study, the in-cylinder gas speed dynamics are derived analytically from the principles of physics in the next subsection.

2.6 In-cylinder Gas Speed Dynamics

In order to evaluate the in-cylinder gas dynamic velocity, the momentum conservation is taken into consideration. Considering the control volume in the diesel engine cylinder shown in Fig. 4, since there is no area change over the infinitesimal length of the engine cylinder, dx, in the engine cylinder, the flow is thus assumed to be quasi one-dimensional flow. The momentum conservation equation states that the net pressure forces plus the wall shear force acting on the control volume surface equal the rate of change of momentum within the control volume plus the net flow of momentum out of the control volume [9]. Thus, the net forces in the control volume shown in Fig. 4, the rate of change of momentum within the control volume, the net flow of momentum across the control volume surface, and the total momentum are investigated in this section, respectively.

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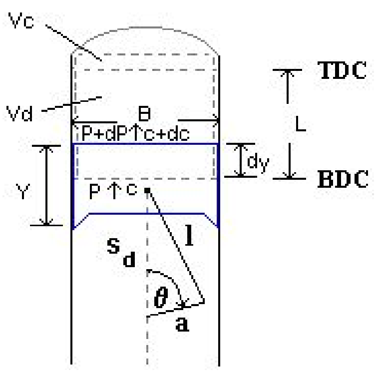

Fig. 4: Control volume in diesel engine cylinder for one-dimensional flow analysis

2.6.1 *The net forces*

The net forces in the control volume shown in Fig. 4 consist of pressure force and shear force. The net pressure force, F_{Press} , can be evaluated as follows:

$$F_{Press} = P_{Cyl}\ A - \left(P_{Cyl} + \frac{\partial P_{Cyl}}{\partial x} dx \right) (A) \tag{132}$$

Where:
A is the cross sectional area of the engine cylinder.
P_{Cyl} is the pressure inside cylinder.

By rearranging Eq. (132), the following follows:

$$F_{Press} = -A \frac{\partial P_{Cyl}}{\partial x} dx \tag{133}$$

The net shear force, F_{Shear} , can be evaluated as follows:

$$F_{Shear} = -\tau_W \left(\pi D\, dx \right) \tag{134}$$

Where:
τ_W is the flow shear stress per unit area,
D is the diameter of the engine cylinder.

By definitions of the flow shear stress and flow friction coefficient, the following follows [9]:

$$\tau_W = \zeta E_{Kin} \tag{135}$$

Where:
ζ is the flow friction coefficient,
E_{Kin} is the kinetic energy per unit area.

The kinetic energy per unit area, E_{Kin}, can be evaluated by definition as follows [9]:

$$E_{Kin} = \rho \frac{c^2}{2} \tag{136}$$

Therefore, combining Eq. (134), (135), and (136) the following follows:

$$F_{Shear} = -\frac{\zeta \rho c^2 \pi D}{2} dx \tag{137}$$

2.6.2 *The rate of change of momentum within the control volume*

The rate of change of momentum within the control volume in the engine cylinder shown in Fig. 4, M_{Moment}, can be evaluated from the fundamental definition of momentum and force as follows [9]:

$$M_{Moment} = \frac{\partial}{\partial t} \left(c\, F_{Moment} \right) \tag{138}$$

Where:
F_{Moment} is the force that generated this momentum.

The force that generated this momentum, F_{Moment}, can be evaluated from its fundamental definition as follows [9]:

$$F_{Moment} = \rho V_{Cyl} \tag{139}$$

Thus, combining Eq. (138) and (139) together leads to the following:

$$M_{Moment} = \frac{\partial}{\partial t} \left(c\ \rho A\, dx \right) \tag{140}$$

2.6.3 *The net flow of momentum across the control volume surface*

The net flow of momentum across the control volume surface in the engine cylinder shown in Fig. 4, M_{Net}, can be evaluated from the fundamental definition of momentum as follows [9]:

$$M_{Net} = \left(\rho + \frac{\partial \rho}{\partial x} dx \right) \left(c + \frac{\partial c}{\partial x} dx \right)^2 (A) - \rho c^2 A \tag{141}$$

Equation (141) can be rearranged as follows:

$$M_{Net} = \left(\rho + \frac{\partial \rho}{\partial x} dx \right) \left(c^2 + \frac{\partial^2 c}{\partial x^2} d^2 x + 2 c \frac{\partial c}{\partial x} dx \right) (A) - \rho c^2 A \tag{142}$$

Equation (142) can be further rearranged as follows:

$$M_{Net} = \left(\rho c^2 + \rho \frac{\partial^2 c}{\partial x^2} d^2 x + 2 \rho c \frac{\partial c}{\partial x} dx + c^2 \frac{\partial \rho}{\partial x} dx + \frac{\partial \rho}{\partial x} dx \frac{\partial^2 c}{\partial x^2} d^2 x + 2 c \frac{\partial c}{\partial x} dx \frac{\partial \rho}{\partial x} dx \right) (A) - \rho c^2 A \tag{143}$$

Equation (143) can be further rearranged as follows:

$$M_{Net} = A \rho c^2 + A \rho \frac{\partial^2 c}{\partial x^2} d^2 x + 2 A \rho c \frac{\partial c}{\partial x} dx + A c^2 \frac{\partial \rho}{\partial x} dx + A \frac{\partial \rho}{\partial x} dx \frac{\partial^2 c}{\partial x^2} d^2 x + 2 A c \frac{\partial c}{\partial x} dx \frac{\partial \rho}{\partial x} dx - \rho c^2 A \tag{144}$$

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By simplifying, ignoring mathematically trivial terms, and rearranging Eq. (144), the following follows:

$$M_{Net} = \frac{\partial}{\partial x}\left(\rho c^2 A\right) dx \tag{145}$$

2.6.4 *The total momentum*

Therefore following from the fundamental definition of the momentum conservation and by combining Eq. (133), (137), (140), and (145) the following follows:

$$-A\frac{\partial P_{Cyl}}{\partial x}dx - \zeta\frac{\rho c^2}{2}\pi D\,dx = \rho A\frac{\partial c}{\partial t}dx + \rho A\frac{\partial c^2}{\partial x}dx \tag{146}$$

By simplifying Eq. (146), the following follows:

$$-A\frac{\partial P_{Cyl}}{\partial x} - \zeta\frac{\rho c^2}{2}\pi D = \rho A\frac{\partial c}{\partial t} + \rho A\frac{\partial c^2}{\partial x} \tag{147}$$

Equation (147) can be rewritten as follows:

$$-A\frac{\partial P_{Cyl}}{\partial x} - \zeta\frac{\rho c^2}{2}\pi D = \rho A\frac{\partial c}{\partial t} + \rho A\left(c\frac{\partial c}{\partial x} + c\frac{\partial c}{\partial x}\right) \tag{148}$$

Equation (148) can be rearranged as follows:

$$-A\frac{\partial P_{Cyl}}{\partial x} - \zeta\frac{\rho c^2}{2}\pi D = \rho A\frac{\partial c}{\partial t} + 2\rho A c\frac{\partial c}{\partial x} \tag{149}$$

Equation (149) can be rearranged as follows:

$$\frac{\partial c}{\partial t} + 2c\frac{\partial c}{\partial x} + \frac{1}{\rho}\frac{\partial P_{Cyl}}{\partial x} + \zeta\frac{c^2}{2A}\pi D = 0 \tag{150}$$

Thus, the in-cylinder gas dynamic velocity can be expressed as follows following from Eq. (150):

$$\dot{c} = -2c\left(\frac{\partial c}{\partial x}\right) - \frac{1}{\rho}\left(\frac{\partial P_{Cyl}}{\partial x}\right) - 2\zeta\frac{c^2}{D} \tag{151}$$

By calling the in-cylinder gas speed analytical formula from the research paper entitled "Diesel Powertrain Intake Manifold Analytical Model" [15] and combining this formula with Eq. (151), the dynamic gas speed can be represented analytically as follows:

$$\dot{c} = -\frac{1}{\rho}\left(\frac{\partial P_{Cyl}}{\partial x}\right) - 2\zeta\frac{c^2}{D} \tag{152}$$

Equation (152) can be further rewritten by calling the in-cylinder gas speed analytical formula from the research paper entitled "Diesel Powertrain Intake Manifold Analytical Model" [15] as follows:

$$\dot{c} = -\frac{1}{\rho}\left(\frac{\partial P_{Cyl}}{\partial x}\right) - \frac{\zeta\,\pi^3 B^4 r_C^{\,2}\,N_m^{\,2}\,a^2}{1800\,D^3}\,Sin^2\,\theta \tag{153}$$

3 DISCUSSION AND CONCLUSION

As the vehicle modelling type that describes the physical phenomena associated with vehicle operation comprehensively based on the principles of physics with explainable mathematical trends, this paper presents an analytical model of the diesel engine of the highly promising diesel powertrains. The compression ratios and temperature ratios on the compression stroke, combustion process, expansion stroke, and exhaust stroke of diesel cycle have been modelled in this study analytically based on the principles of physics. Equation (39) provides an analytical model of the pressure ratio over the compression stroke of diesel cycle. It shows analytically the relation between the angle of rotation of the crankshaft and the pressure ratio on the compression stroke of diesel cycle. The temperature ratio over the combustion process of diesel cycle has been modelled analytically as well based on the principles of physics. Equation (62) shows analytically the relation between the angle of rotation of the crankshaft and the temperature ratio on the combustion process of diesel cycle. Equation (86) provides an analytical model of the pressure ratio over the expansion stroke of diesel cycle. It shows analytically the relation between the angle of rotation of the crankshaft and the pressure ratio on the expansion stroke of diesel cycle. The temperature ratio over the exhaust process of diesel cycle has been modelled analytically as well based on the principles of physics. Equation (110) shows analytically the relation between the angle of rotation of the crankshaft and the temperature ratio on the exhaust process of diesel cycle.

The study has derived analytically as well the states throughout diesel cycle from the first principles of physics. The pressure, temperature, and specific volume at state 1 on diesel cycle have been analytically modelled in Eq. (111), (112), and (114), respectively. State 2 on diesel cycle has been fully defined analytically as well. The pressure, temperature, and specific volume at state 2 on diesel cycle have been analytically modelled in Eq. (115), (117), and (118), respectively. At state 3 on diesel cycle, the pressure, temperature, and specific volume have been analytically modelled in Eq. (121), (122), and (131), respectively. State 4 on diesel cycle has been fully defined analytically as well. The pressure, temperature, and specific volume at state 4 on diesel cycle have been analytically modelled in Eq. (124), (123), and (126), respectively.

Finally, the paper has elucidated the in-cylinder gas speed dynamics in diesel engines. Equation (153) presents an analytical model of the in-cylinder gas speed dynamics as a function of gas pressure inside cylinders, gas density, flow friction coefficient, and cylinder bore diameter. The developed analytical models in this study address flaws in models presented in key references in this research area. All of the analytical models presented in this study have been derived step by step from the principles of physics as a way of validating these models. The presented models in this research work can help in analyzing analytically with explainable mathematical trends the performance of supercharged diesel engines with respect to both of the transient response and steady state response. There are two key advantages of these developed analytical models: (1) Widely valid model that is not restricted to a specific dataset; (2) Can be helpful in developing and assessing diesel powertrain technologies. This research serves the ITS by facilitating the analytical modelling of diesel powertrain fuel consumption and exhaust emissions rates which are of prime interest in diesel technologies development. This study ex-

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hibits research on further validating experimentally the developed analytical models.

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